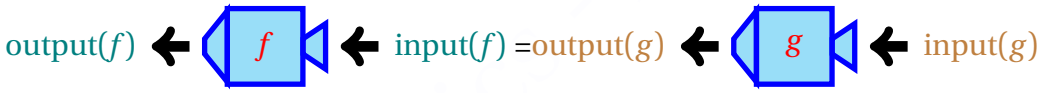
Worksheet Two **MATH 104** Name:_____

WORK ON THIS ASSIGNMENT IN GROUP OF 2-4. TURN IN YOUR WORK INDIVIDUALLY IN CLASS. YOU CAN USE YOUR NOTES FOR THIS ASSIGNMENT.

1.4: Function Operations and Composition

- Let f and g be two functions, then $f+g$ (sum of two functions) is a function whose domain is intersection of the two domains such that $(f+g)(x) = f(x) + g(x)$.
- $f-g$ (Difference of two functions) , $f \cdot g$ (Product of two functions) are defined similarly as $f+g$.
- $\frac{f}{g}$ (The quotient of two functions) is defined to be $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$ for x in the intersection of the two domains where $g(x) \neq 0$.
- The composite function $f \circ g$ ($f \underbrace{\circ}_{\text{reads "of"}} g$) is defined to be $(f \circ g)(x) = f(g(x))$ for all x in the domain of g and $g(x)$ in the domain of f. So the output of g is the input of f.



$$f(g(x))$$

Finding the composition of two functions

- Identify the outer and inner function. For example in $f \circ g$, f is the outer and g is the inner function.
- Write the outer and write big parentheses whenever you see the independent variable.
- Write the inner function in every parentheses.
- The domain of a composite function, $f \circ g$ is all input values x in the domain of g for which $g(x)$ is in the domain of f. The process of finding domain when the two functions are elementary functions may involve finding the simplified rule of $f \circ g$ and its domain, and then intersecting this domain with the domain of g.
- Section 1.3 Review: Suppose velocity $v = c$ is constant; the distance traveled is $x = vt$, where t is time.

1. A store is offering a 15% discount on all items. You have a coupon worth \$10 off any item more expensive than \$10. Let x represent the price, in dollars.

   (a) Suppose only 15% discount applies. Express the final cost of an item as a function, f, of its regular price, x (f in terms of x). What is the domain of f?

   (b) Suppose that only \$10 off coupon applies. Express the final cost of an item as a function, g, of its regular price, x (g in terms of x). What is the domain of g?

   (c) What is the meaning of $f \circ g$ in this problem?

   (d) What is the meaning of $g \circ f$ in this problem?

   (e) Compute both $f \circ g$ and $g \circ f$ and find their domains.

   (f) How much will you pay for a \$150 item if you use \$10 off coupon first and then use 15% discount? Would you save more if you reverse the order of the coupons?

   (g) If you have another coupon that is 20% off any item, what is the final cost of an item priced at x dollars after using this coupon? Express as function, h, in terms of x.

   (h) Find $f \circ h$ and $h \circ f$.

2. Water is being pumped into a cylindrical tank at a constant rate. The diameter of the tank is 10 meters and its height is 7 meters. The water level, h, is rising at a constant rate of $0.2\,\frac{m}{min}$ and when we start filling the tank, at time $t = 0$, the water level is already at 2 m.

   (a) What type of the function has a constant rate of change?

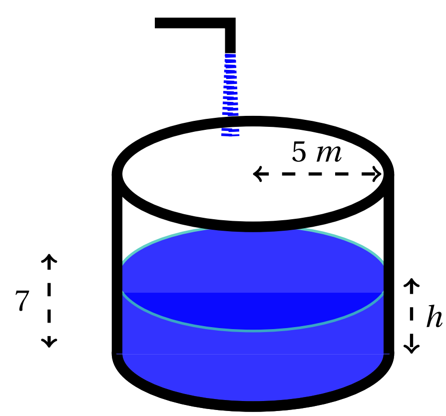
   (b) Find the water level, h, as a function of time in minutes, t.

   (c) How long does it takes to fill out the tank?

   (d) What is the domain of $h(t)$?

   (e) Find the volume of water in the cylinder as a function of height in meters, h.

   (f) Find the volume of the water as a function of time, t.



3. Given $f(x) = x^2 + 4$ and $g(x) = \sqrt{x} - 4$, find the value of $(f + g)(9)$.

4. Given $f(x) = x^2 + 4$ and $g(x) = \sqrt{x} - 4$, find the value of $(g - f)(9)$.

5. Given $f(x) = x^2 + 4$ and $g(x) = \sqrt{x} - 4$, find the value of $(\frac{g}{f})(a)$.

6. Given $f(x) = x^2 + 4$ and $g(x) = \sqrt{x} - 4$, find the value of $2f(1)$.

7. Given $f(x) = x^2 + 4$ and $g(x) = \sqrt{x} - 4$, find the value of $g(f(\sqrt{2}))$.

8. Given $g(x) = 9x + \dfrac{2}{x-1}$ and $f(x) = 1 + 2x$, find $f(g(x))$.

9. Given $g(x) = 5x^2 - 2$ and $f(x) = \sqrt{x} + 1$, find $f(g(x))$.

10. Values of the functions f and g are given in the table to the right. (I) The value of $g(f(4))$ is

x	$f(x)$	$g(x)$
1	3	3
2	1	4
3	4	2
4	2	1

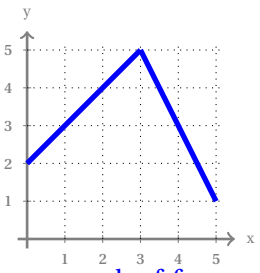
(A) 1 (D) 4

(B) 2 (E) 8

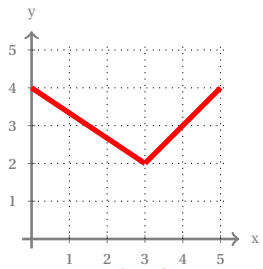
(C) 3 (F) None of these

(II) Average rate of $g(x)$ over the interval $(1,3)$ is

11. Graphs of functions f and g are given to the right. What is $(g \circ f)(4)$? *(Circle only one)*



graph of f



graph of g

(A) 2 (C) 4

(B) 3 (D) 5

INDIVIDUAL WORK

UPLOAD TO CANVAS OR SUBMIT IN CLASS BEFORE DUE DATE. DISCUSSING THESE QUESTIONS IN YOUR GROUP IS ENCOURAGED BUT MAKE SURE YOU ARE TURNING IN YOUR OWN WORK.

12. *(1 point)* Given $f(x) = x^2 + x^{\frac{1}{2}}$ and $g(x) = x^4$, find $f(g(x))$.

13. *(1 point)* Given $f(x) = \dfrac{3}{x} - x$ and $g(x) = \dfrac{x}{3} + x$, find $f(g(x))$.

14. *(1 point)* Given $f(x) = x^2 + 6$ and $g(x) = \sqrt{x} - 6$, find the value of $(f - g)(16)$.

15. *(1 point)* Given $f(x) = x^2 + 6$ and $g(x) = \sqrt{x+6} - 6$, find the value of $g(f(x))$.

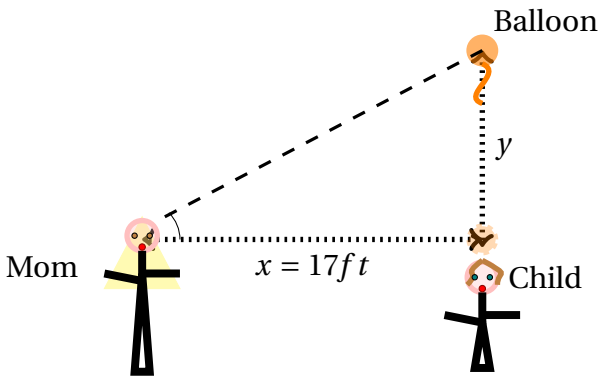
16. *(2.5 point)* Given $f(x) = x^2 + 5x - 5$ and $g(x) = x - c$, find $\dfrac{f(g(x)) - f(x)}{c}$ and simplify.

17. *(1.5 points)* Albert lets go of his balloon and the balloon starts rising vertically at a **constant speed** of 6 feet per second. Mom who is standing 17 feet from Albert is looking directly at the balloon, which was originally in her horizontal line of sight, stays frozen in her place and keeps looking straight at the balloon as it is rising. Let y be the vertical distance between Albert and the balloon, in feet.

(a) Express the distance between the mom and the balloon, D, as a function of y (in terms of y).

(b) Express y as a function of t, time in seconds (in terms of t).

(c) Express D as a function of t (in terms of t).



Example Videos:

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2. https://mediahub.ku.edu/media/t/1_t37r3h2y

The Related Videos

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